

Diagnostics of an electron beam of a linear accelerator using coherent transition radiation

Yukio Shibata, Toshiharu Takahashi, Toshinobu Kanai, Kimihiro Ishi, and Mikihiro Ikezawa
Research Institute for Scientific Measurements, Tohoku University, Katahira, Aoba-ku, Sendai 980, Japan

Juzo Ohkuma, Shuichi Okuda, and Toichi Okada

Radiation Laboratory, The Institute of Scientific and Industrial Research, Osaka University, Mihogaoka, Ibaraki, Osaka 567, Japan

(Received 30 July 1993; revised manuscript received 28 February 1994)

The three-dimensional distribution and the divergence of electrons in a bunch have been determined for a short-bunch beam of an L -band linear accelerator by the observation of coherent transition radiation at millimeter and submillimeter wavelengths. The transverse size of the beam and its emittance are derived as 22 ± 4 mm in diameter and $(4.0 \pm 2.0) \times 10^2$ mm mrad. The longitudinal distribution of electrons in the bunch is found to be a triangular shape with a bunch length (full width at half maximum) of 8.5 ± 1.0 mm (28 ps). An advantage of this method in determining the longitudinal distribution function of electrons is discussed in comparison with the conventional method using a streak camera.

PACS number(s): 41.85.Qg, 41.60.-m, 41.75.Ht

I. INTRODUCTION

Beam diagnostics is an important aspect of many research areas, such as free-electron lasers, synchrotron radiation from storage rings, and electron-positron colliders in high energy physics, where electron beams of low emittance are used. Hitherto, Cherenkov radiation or synchrotron radiation has been observed with a streak camera to determine the longitudinal length of a bunch of charged particles. The transverse shape of the beam has been characterized with other apparatus such as wire scanners, photoemission devices, or video cameras which observe either a fluorescent screen or transition radiation in the visible region. In the present experiment, spectroscopic observation of coherent transition radiation (TR) is explored as an alternative method which can determine not only the longitudinal and transverse distributions of electrons in the bunch but also the divergence of the bunch.

Coherent radiation is emitted from bunched electrons. The size of the bunch is shorter than or nearly equal to the wavelengths of the radiation. The coherence effect of synchrotron radiation was studied theoretically [1,2] and experimentally in the far-infrared region [3-5]. Using an S-band linear accelerator, Ishi *et al.* [5] observed a spectrum of coherent synchrotron radiation in a wide wavelength range and confirmed the relation between its spectrum and the longitudinal distribution of the electrons in a bunch. The analysis of the spectrum gave a longitudinal bunch shape with a precision of the order of the wavelength of the observed radiation.

TR is emitted when charged particles pass through a boundary of two media with different dielectric constants. Recently, coherent TR from bunched electrons passing through a metallic foil in vacuum has been observed at millimeter and submillimeter wavelengths [6-9] and its properties were investigated by Shibata *et al.* [8,9]. Their results suggest that we can derive information about bunched electrons from analysis of the spectrum and the angular distribution of coherent TR.

In this paper, we formulate the coherence effect in TR to determine the distribution function of electrons in a bunch and divergence of the beam. On the basis of the formulation, we have analyzed a picosecond bunch of electrons having a charge of 26 nC and an energy of 28 MeV from an L -band linear accelerator. The results are compared with those obtained by other conventional methods.

II. METHOD OF ANALYSIS

A. Theory of coherent transition radiation

We consider TR emitted from a bunch of N electrons passing through a metallic foil. The trajectory of the center of the bunch is taken as the z axis and an observation point in the direction of angle θ from the z axis is assumed to be far from the foil (see Fig. 1). Then, the radiation field at the observation point is given by a superposition of plane waves emitted from the electrons in the bunch as

$$\mathbf{E}_T = \sum_{j=1}^N \frac{\mathbf{k}_j \times (\mathbf{k}_j \times \mathbf{V}_j)}{|\mathbf{k}_j \times (\mathbf{k}_j \times \mathbf{V}_j)|} E(\lambda, \theta_j) \exp[i(\mathbf{k}_j \cdot \mathbf{X}_j)/\lambda], \quad (1)$$

where \mathbf{X}_j is the position vector of the j th electron relative to the bunch center, \mathbf{V}_j its velocity, \mathbf{k}_j the wave vector directed from the electron to the observation point, and θ_j is the angle between \mathbf{V}_j and \mathbf{k}_j .

Usually N is a large number and the summation in Eq. (1) can be replaced by an integral [1,2]. Since the observation point is far from the radiator, the vector \mathbf{k}_j is well approximated by $2\pi\mathbf{n}/\lambda$ where the unit vector \mathbf{n} is directed from the center of the bunch to the observation point. We assume that the position of the electron has no correlation with its direction of motion. Then, the intensity of coherent TR is

$$P(\lambda) = |\mathbf{E}_T|^2 = Q(\lambda) N^2 \chi f, \quad (2)$$

where $Q(\lambda)$ is the intensity of TR emitted from a single

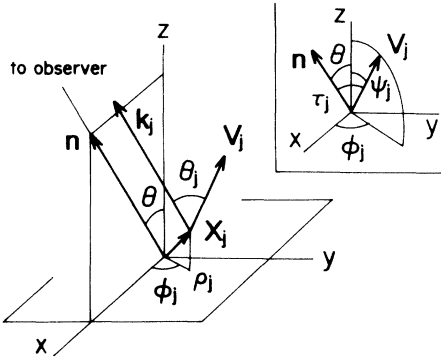


FIG. 1. Geometry illustrating transition radiation from a bunch. \mathbf{X}_j is the position vector of the j th electron and \mathbf{V}_j is its velocity. An observation point and the trajectory of the center of the bunch (z axis) define the xz plane, and ϕ is measured from the x axis. The inset shows the angles that define the direction of motion of the electron in the bunch.

electron. A bunch form factor f is given by the square of a structure factor F which is defined by the Fourier transform of a distribution function $S(\mathbf{X})$ of the electronic density in the bunch [1–6]

$$f = |F|^2 = \left| \int S(\mathbf{X}) \exp[i2\pi(\mathbf{n} \cdot \mathbf{X})/\lambda] d\mathbf{X} \right|^2. \quad (3)$$

A divergence factor χ is derived from the addition of the electric vectors of radiation at the observation point as

$$\chi = \left[\int \mathbf{n} \times (\mathbf{n} \times \mathbf{V}) / (|\mathbf{V}| \sin \tau) G(\mathbf{V}) d\mathbf{V} \right]^2, \quad (4)$$

where τ is the angle between the vector \mathbf{V} and \mathbf{n} , and $G(\mathbf{V})$ is a velocity distribution of the electron in the bunch. The speed of an electron having an energy several tens of MeV or more is well approximated by that of light in vacuum. Hence, $G(\mathbf{V})$ expresses only a distribution function of the direction of motion of the electrons. The distribution functions $S(\mathbf{X})$ and $G(\mathbf{V})$ are normalized to unity; $\int S(\mathbf{X}) d\mathbf{X} = 1$, and $\int G(\mathbf{V}) d\mathbf{V} = 1$.

When the bunch size is much less than the wavelength and the beam has no divergence, both of the factors f and χ have the maximum value of unity (coherent limit).

We assume that $S(\mathbf{X})$ has axial symmetry and adopt a cylindrical coordinate system (ρ, ϕ, z) , where ϕ is measured from the x axis (Fig. 1). The function $S(\mathbf{X})$ is given by a product of a longitudinal distribution function $h(z)$ and a transverse one $g(\rho)$. The form factor f is expressed by a product of the longitudinal bunch form factor f_L and the transverse one f_T [10] as $f = f_L f_T$. Since $(\mathbf{n} \cdot \mathbf{X})$ is expressed as $(\rho \sin \theta \cos \phi + z \cos \theta)$ for radiation directed at an angle θ from the beam axis, f_L and f_T are as follows:

$$f_L = |F_L|^2 = \left| \int h(z) \exp[i2\pi z \cos \theta / \lambda] dz \right|^2, \quad (5)$$

$$\begin{aligned} f_T &= |F_T|^2 \\ &= \left| \int \int g(\rho) \exp[i2\pi \rho \sin \theta \cos \phi / \lambda] \rho d\rho d\phi \right|^2 \\ &= \left| 2\pi \int g(\rho) J_0(2\pi \rho \sin \theta / \lambda) \rho d\rho \right|^2, \end{aligned} \quad (6)$$

where J_0 is the Bessel function of the zeroth order.

In the case of a relativistic electron, most of the flux of the TR is emitted within a narrow range of the direction angle θ , which satisfies the inequality $\theta^2 \ll 1$. We can consider the longitudinal bunch form factor of Eq. (5) to be independent of the direction angle θ .

To examine the divergence factor of Eq. (4), we use a polar coordinate system (V, ψ, ϕ) in a velocity space as shown in the inset of Fig. 1; the polar angle ψ is the angle between the direction of motion of the electron and the z axis. The distribution $G(\mathbf{V}) d\mathbf{V}$ is expressed as $G(\psi, \phi) \sin \psi d\psi d\phi$. Under the assumption of the axial symmetry of the distribution of the direction of motion, the divergence factor is given by

$$\chi = \left[\int \int \frac{\cos \theta \sin \psi \cos \phi - \sin \theta \cos \psi}{\sin \tau} G(\psi) \sin \psi d\psi d\phi \right]^2. \quad (7)$$

When the distribution functions $h(z)$, $g(\rho)$, and $G(\psi)$ are known, we can calculate the spectrum and the angular distribution of coherent TR, using Eqs. (5)–(7). In the inverse way, from the observation of properties of coherent TR, we can derive information on the distribution of the electrons and its divergence. This is the basis of the method explored in the present experiment.

In a special case, when the transverse size of the bunch is less than the wavelength and the divergence of the beam is small enough, the factors f_T and χ are approximated by unity. In this case, using Eq. (2), we can obtain the longitudinal bunch form factor f_L from the observed spectrum of the coherent radiation $P(\lambda)$, because the quantity $Q(\lambda)$ is known from calculation on the experimental conditions. Then, the longitudinal distribution function of the electrons is derived from the inverse Fourier transform of the form factor as follows:

$$h(z) = \int f_L^{1/2} \cos(2\pi z / \lambda) d(1/\lambda). \quad (8)$$

B. The structure factor and divergence factor for Gaussian distributions

For numerical calculations in Sec. IV B, some equations for the Gaussian distribution are derived here.

When the longitudinal distribution is given by the Gaussian function $h(z) = \exp(-z^2/H^2)/(\pi^{1/2}H)$, we get the longitudinal structure factor

$$F_L = \exp[-(\pi H \cos \theta / \lambda)^2]. \quad (9)$$

For the transverse distribution $g(\rho) = \exp(-\rho^2/\sigma^2)/(\pi\sigma^2)$, the transverse structure factor is

$$F_T = \exp[-(\pi\sigma \sin \theta / \lambda)^2]. \quad (10)$$

For the Gaussian distribution of direction of motion of electrons $G(\psi) = \exp(-\psi^2/\Omega^2)/(\pi\Omega^2)$, the divergence factor of Eq. (7) is expressed as

$$\chi = \left\{ \frac{2\theta^2}{\pi\Omega^2} \int_0^{\pi/(2\theta)} x[(1-x)K(y) + (1+x)E(y)] \times \exp \left[- \left[\frac{\theta}{\Omega} \right]^2 x^2 \right] dx \right\}^2, \quad (11)$$

where $K(y)$ and $E(y)$ are the complete elliptic integrals of the first and second kinds, whose modulus is given as a function of the variable of the integral in Eq. (11) as $y = 2x^{1/2}/(1+x)$:

$$K(y) = \int_0^{\pi/2} (1 - y^2 \sin^2 \omega)^{-1/2} d\omega, \quad (12)$$

$$E(y) = \int_0^{\pi/2} (1 - y^2 \sin^2 \omega)^{1/2} d\omega. \quad (13)$$

Each of the factors F_L , F_T , and χ has only one free parameter: H , σ , and Ω , respectively.

III. EXPERIMENT

A. Experimental setup

We have used a single-bunch beam from the 38-MeV L-band Electron Linear Accelerator of the Institute of Scientific and Industrial Research, Osaka University. The experimental setup is shown schematically in Fig. 2. The linear accelerator is equipped with a high-current electron gun, a specific prebuncher, ordinary bunchers, and a bunch compressor [12–14]. The characteristics of the single-bunch beam were as follows: The total charge of electrons in a bunch was 26 nC (1.6×10^{11} electrons), the energy of electrons and the energy spread were 28 MeV and 2.9%, and the repetition rate of the bunch was 120 Hz.

The electron beam passed through a 15- μm -thick aluminum foil P in Fig. 2 and a 15- μm -thick aluminum foil-mirror $M1$ in an evacuated chamber. Forward TR was emitted from the Al foil and was reflected by the mirror $M1$ to a collecting mirror $M2$ with an acceptance angle of 60 mrad. The foil-mirror $M1$ also emitted backward TR towards the mirror $M2$. The emission length L , i.e., the distance between the points P and $M1$, was 165 mm. The TR collected by $M2$ was guided to a grating-type far-infrared spectrometer which was equipped with several gratings and appropriate filters [5]. The radiation was detected by a liquid-He-cooled Si bolometer.

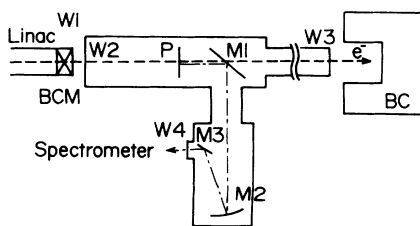


FIG. 2. Schematic diagram of the experiment. $W1$, $W2$: Ti-foil window; $W3$: Al window; $W4$: quartz window; P : Al foil; $M1$: Al-foil mirror; $M2$: spherical mirror; $M3$: plane mirror; BCM: beam current monitor; BC: beam catcher. The trajectory of the beam is shown by the dashed line, where Linac is the linear accelerator.

The angular distribution of coherent TR was observed by rotating the mirror $M1$ around a vertical axis.

Unwanted radiation, such as forward TR emitted from an upstream Ti foil window $W2$ and backward TR from the Al foil P , was carefully blocked out so as not to propagate into the measuring system. The spectral sensitivity of the measuring system was calibrated by blackbody radiation from a graphite cavity of 1200 K. Observational accuracy of the absolute intensity of TR was estimated to be within a factor of 1.5.

B. Analysis of a two-radiator system

In Sec. II A, we have considered coherent TR from a single boundary. The TR intensity observed in the experiment is given as the superposition of the forward TR from the Al foil (P in Fig. 2) and the backward TR from the foil-mirror $M1$ [9,11]. In the case of this two-radiator system, coherent TR of Eq. (2) is expressed as

$$P(\lambda) = q(\lambda) R N^2 \chi f_L |F_{Tf} - F_{Tb} \exp(-i\Phi)|^2, \quad (14)$$

$$q(\lambda) = \frac{\alpha \beta^2 \sin^2 \theta \cos^2 \theta}{\pi^2 \lambda (1 - \beta^2 \cos^2 \theta)^2} |\zeta|^2, \quad (15)$$

where α and β are the fine structure constant and the ratio of the velocity of the electron to the light velocity in vacuum, respectively, ζ is a factor depending on the dielectric constant of the Al foil, and R is the reflectivity of the foil-mirror. In the present experiment, R and $|\zeta|$ are well approximated by unity [9]. The phase difference Φ between the forward TR and the backward TR is given by $\Phi = L/Z_f$, where L is the emission length and Z_f is the formation length, $Z_f = \beta \lambda / [2\pi(1 - \beta \cos \theta)]$.

The transverse structure factors F_{Tf} and F_{Tb} should be evaluated at the Al foil position and at the mirror position, respectively. The value of the beam parameter σ in Eq. (10) at the mirror position, which is denoted as σ_b , is related to the one at the foil position σ_f by $\sigma_b - \sigma_f = L\Omega$, where Ω is the divergence parameter in Eq. (11).

IV. RESULTS AND DISCUSSION

A. Properties of the coherent transition radiation

The observed spectrum is shown by the solid curve in Fig. 3. The dependence of the spectral intensity (P) on the beam current (I) was measured at $\lambda = 5$ mm. The result shows that the power index defined by $\Delta \ln(P) / \Delta \ln(I)$ is 2.3 ± 0.3 . This result confirms that we have observed the coherent radiation.

The dashed line in Fig. 3 shows the theoretical intensity of incoherent TR calculated for the experimental condition. Compared with the calculation, the observed intensity at $\lambda = 3$ mm was enhanced by a factor of 5×10^6 . This enhancement is due to the coherence effect.

The angular distributions of the coherent TR observed at $\lambda = 1, 2, 3$, and 4 mm are shown by solid curves in Fig. 4. Each of the distributions is nearly symmetric with respect to the beam axis, $\theta = 0$, and has two peaks, except for one at $\lambda = 1$ mm. The results show that TR is emitted

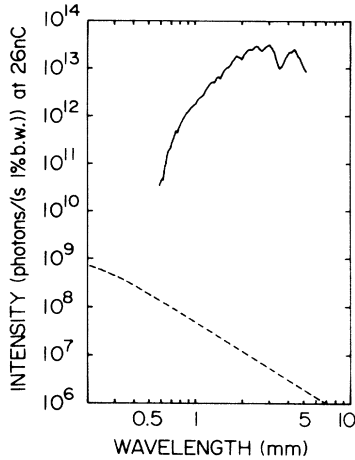


FIG. 3. Spectrum of coherent TR. The solid curve shows the experimental results. The intensity is given in units of photon numbers per bunch of 26 nC per bandwidth of 1%, i.e., per $\Delta\lambda/\lambda$ of 0.01. The dashed curve shows the theoretical intensity of the incoherent TR.

in a cone and that its divergence increases as the wavelength increases [8,9]. In Fig. 4 the conical structure was not resolved at $\lambda=1$ mm, because the angular resolution of the measuring system was as low as 52 mrad.

B. Beam diagnostics

The factors f_T and χ depend on the direction angle θ , while f_L depends little on θ . Therefore we first determined the diameter of the beam and its divergence from analysis of the angular distribution of coherent TR shown in Fig. 4, assuming that the distribution functions $g(\rho)$ and $G(\psi)$ were Gaussian. Then the longitudinal distribution of the electrons, f_L , is derived from analysis of the

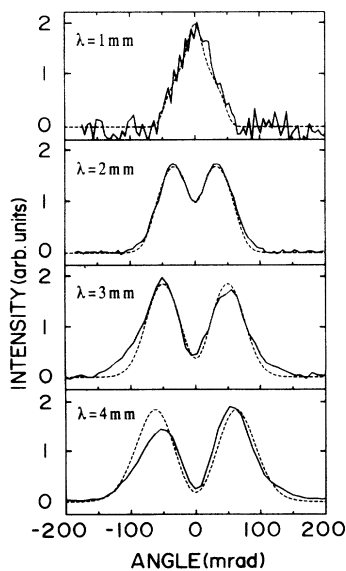


FIG. 4. Angular distribution of coherent TR at $\lambda=1, 2, 3$, and 4 mm. Dashed curves show those distributions calculated on an assumption of a Gaussian bunch whose diameter and divergent angle at the point P in Fig. 2 are 22 mm and 36 mrad.

spectrum of the radiation.

Varying the parameters σ_f and Ω , we calculated the angular distribution of the radiation from Eqs. (10), (11), (14), and (15) so as to fit the observation. In Fig. 4, the dashed curves show the calculated distributions that agree best with the observations. The results are that the diameter of the beam ($1.67\sigma_f$) at half maximum is 22 ± 4 mm at the Al-foil position, and the half angle of the divergence (0.83Ω) is 36 ± 17 mrad. From these results, the emittance of the beam is calculated to be $4.0\pm 2.0\times 10^2$ mm mrad.

The accuracy of the measurement of the divergent angle will be improved if the emission length L is increased and the angular resolution of the measuring system is made higher.

The spectral shape of the longitudinal bunch form factor f_L is obtained from the observed spectrum in Fig. 3 with the calculation by Eq. (14). We have used the beam diameter and the divergence determined above. The result is shown by the solid curve in Fig. 5.

The form factor in Fig. 5 increases with the wavelength, and is nearly proportional to λ^4 . Since its value is much less than unity, we were not able to obtain reliable longitudinal distribution of electrons from the inverse Fourier transform of the form factor. Hence we calculated the form factor f_L assuming a functional form of $h(z)$, and searched for the best function $h(z)$ to fit the observation.

At first, we calculated the form factor assuming a Gaussian function. The calculation with a bunch length [full width at half maximum (FWHM)] of 7.2 mm is shown by the solid circles in Fig. 5, as an example. The gradient of the calculated form factor is much steeper than the observed one. When the bunch length (FWHM) is varied, the form factor only shifts horizontally in Fig. 5. Hence a simple Gaussian distribution of electrons disagrees with the observation.

If we expand $h(z)$ as a polynomial function of z , the contribution to the form factor from a term z^n is calculat-

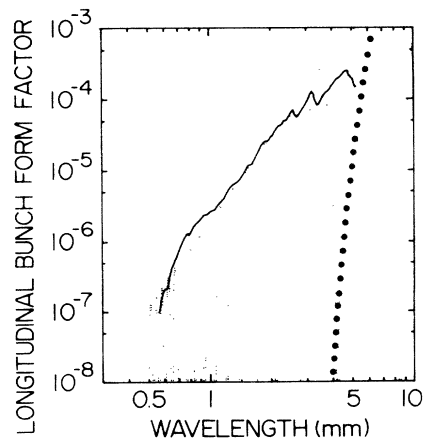


FIG. 5. Longitudinal bunch form factor derived from the observed spectrum. The solid circles show the form factor calculated on an assumption of a Gaussian bunch with a length (FWHM) of 7.2 mm, and the dotted curve that of a triangular distribution with a bunch length (FWHM) of 8.5 mm.

ed by Eq. (5) as

$$f_L = \lambda^{2(n+1)} U(z_b/\lambda) + \sum_{m=0}^{2n+1} \lambda^m V_m(z_b, \lambda), \quad (16)$$

where z_b is the length within which the integration of Eq. (5) is carried out, U is a trigonometric function of $2\pi z_b/\lambda$, and V_m is a polynomial function of z_b , $\sin(2\pi z_b/\lambda)$, and $\cos(2\pi z_b/\lambda)$. The function U oscillates rapidly with wavelength λ , but the envelope of the oscillation is roughly proportional to $\lambda^{2(n+1)}$.

The observed form factor in Fig. 5 is roughly proportional to λ^4 . The main contribution should be derived from the term of $n=1$. In the first approximation, we searched for an appropriate form of $h(z)$ as a linear function of z . As a result of the curve fitting to the experimental form factor, we have obtained the following distribution function: $h(z) = (1/z_b)(1 - |z|/z_b)$ with $z_b = 8.5 \pm 1.0$ mm. The function represents a triangular distribution of electrons with a bunch length (FWHM) of 8.5 mm. The form factor corresponding to the distribution is expressed as $f_L = [\sin(\pi z_b/\lambda)/(\pi z_b/\lambda)]^4$, and is shown by the dotted curve in Fig. 5. The form factor oscillates with decreasing amplitude toward short wavelengths, and the envelope is proportional to λ^4 . In the experiment we have observed with a streak camera that the distribution of electrons fluctuated from bunch to bunch. We therefore believe that the fluctuation probably smeared the oscillatory structure of the spectrum.

The accuracy of the observation of TR intensity affects the accuracy of determining the distribution function. In this experiment the uncertainty of the factor of 1.5 of the TR intensity makes no change in the functional form but it brings about a change of 11% in the bunch length.

C. Comparison of present method with conventional ones

To confirm the beam diameter determined by coherent TR, the transverse size of the beam was measured by a film dosimeter located at the Al-foil position. The film showed a circular pattern with a diameter at the half maximum of 18 ± 1 mm. This result agrees well with the beam diameter derived from TR (22 ± 4 mm) within the accuracy.

To confirm the longitudinal distribution of electrons in the bunch, the single-bunch beam was led from the linear accelerator into air through the 30- μ m-thick titanium window $W1$ in Fig. 1; a plane mirror was located on the trajectory of the beam at the position of 27 cm downstream from the window to observe a duration of Cherenkov radiation in the visible region with a streak camera. An average of ten bunches is shown in Fig. 6 by solid circles; the bunch length (FWHM) was found to be 24 ps (7.2 mm in length).

We have analyzed the result observed with the streak camera on the basis of theory of ordinary Cherenkov radiation and have ignored forward TR emitted from the window and backward TR from the mirror for the following reasons. The longitudinal length of the bunch was much longer than the wavelength in the visible region. The coherence effect has accordingly no influence on the observed radiation. The path length of 27 cm in air at at-

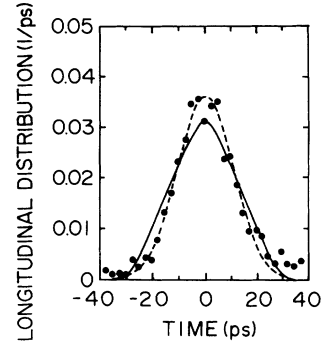


FIG. 6. Longitudinal bunch shape of a single-bunch beam (solid circles) obtained with a streak camera from a measurement of Cherenkov radiation in air. The solid curve shows the bunch shape calculated from the triangular distribution with a bunch length (FWHM) of 8.5 mm (28.3 ps). The dashed curve shows the Gaussian bunch with a length (FWHM) of 7.2 mm (24 ps). Each of the distributions is normalized to unity.

mospheric pressure is sufficiently long to observe fully developed Cherenkov radiation in the visible region; hence the TR is much weaker than the Cherenkov radiation [15].

The time resolution of the streak camera including the aberration of the optical system and the dispersion of the refractive index of air was evaluated to be 7 ps. The triangular distribution function of electrons with the bunch length (FWHM) of 8.5 mm (28.3 ps) was convoluted with a gate function of the triangular shape whose bandwidth (FWHM) was 7 ps. The result is shown by the solid curve in Fig. 6. The triangular distribution agrees well with the result of the streak camera.

With regard to the longitudinal distribution, it should be noted that the information obtained from the spectrum of coherent TR is more detailed than that from the streak camera. To make this point clear, we have assumed a Gaussian distribution with a bunch length of 7.2 mm. The bunch shape convoluted with the gate function of 7 ps is shown by the dashed curve in Fig. 6. This curve agrees very well with the observed shape and resembles that of the triangular distribution. In other words, we can hardly distinguish the Gaussian distribution from the triangular one by the method of the streak camera. However, the form factor calculated with the Gaussian distribution, which is shown by solid circles in Fig. 5, differs drastically from that of the triangular distribution. This result indicates that the spectrum of coherent TR gives more specific information on the functional form of the distribution of electrons compared with the conventional method using the streak camera.

In the conventional method, the bunch length is measured in time domain with a streak camera. Accuracy of the method in bunch shape is limited by the time resolution of the streak camera, and it is difficult to attain a resolution of 0.1 ps. The method developed in this paper, on the other hand, is based on the measurement of the coherent radiation in frequency domain. The bunch shape is determined in the precision of the minimum wavelength of an observed spectrum of the radiation. The method based on the analysis of the coherence effect

in the radiation requires observation of the spectrum in a wavelength range covering radiation from the length of the bunch to roughly an order of magnitude smaller. Since the optical spectrum in the infrared or visible region is more easily observed than that in the far-infrared region, the method will be applicable to a short bunch a picosecond to a femtosecond in duration with high resolution.

ACKNOWLEDGMENTS

We thank the staff at the Radiation Laboratory, ISIR Osaka University for their support during the experiment. This work was supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

-
- [1] J. S. Nodvick and D. S. Saxon, *Phys. Rev.* **96**, 180 (1954).
 - [2] C. J. Hirschmugl, M. Sagurton, and G. P. Williams, *Phys. Rev. A* **44**, 1316 (1991).
 - [3] T. Nakazato, M. Oyamada, N. Niimura, S. Urasawa, O. Konno, A. Kagaya, R. Kato, T. Kamiyama, Y. Torizuka, T. Nanba, Y. Kondo, Y. Shibata, K. Ishi, T. Ohsaka, and M. Ikezawa, *Phys. Rev. Lett.* **63**, 1245 (1989).
 - [4] Y. Shibata, K. Ishi, T. Ohsaka, H. Mishiro, T. Takahashi, M. Ikezawa, Y. Kondo, T. Nakazato, M. Oyamada, N. Niimura, S. Urasawa, R. Kato, and Y. Torizuka, *Nucl. Instrum. Methods Phys. Res. Sect. A* **301**, 161 (1991).
 - [5] K. Ishi, Y. Shibata, T. Takahashi, H. Mishiro, T. Ohsaka, M. Ikezawa, Y. Kondo, T. Nakazato, S. Urasawa, N. Niimura, R. Kato, Y. Shibasaki, and M. Oyamada, *Phys. Rev. A* **43**, 5597 (1991).
 - [6] Y. Shibata, K. Ishi, T. Takahashi, F. Arai, M. Ikezawa, K. Takami, T. Matsuyama, K. Kobayashi, and Y. Fujita, *Phys. Rev. A* **44**, 3449 (1991). See the Note added in proof in this reference.
 - [7] U. Happek, A. J. Sievers, and E. B. Blum, *Phys. Rev. Lett.* **67**, 2962 (1991).
 - [8] Y. Shibata, K. Ishi, T. Takahashi, T. Kanai, M. Ikezawa, K. Takami, T. Matsuyama, K. Kobayashi, and Y. Fujita, *Phys. Rev. A* **45**, 8340 (1992).
 - [9] Y. Shibata, K. Ishi, T. Takahashi, T. Kanai, F. Arai, S. Kimura, T. Ohsaka, M. Ikezawa, Y. Kondo, R. Kato, S. Urasawa, T. Nakazato, S. Niwano, M. Yoshioka, and M. Oyamada, *Phys. Rev. E* **49**, 785 (1994).
 - [10] L. A. Vardanyan, G. M. Garibian, and C. Yang, *Izv. Acad. Nauk Arm. SSR Fiz.* **10**, 350 (1975).
 - [11] L. W. Wartski, S. Roland, J. Lasalle, M. Bolore, and G. Filippi, *J. Appl. Phys.* **46**, 3644 (1975).
 - [12] M. Kawanishi, K. Hayashi, T. Okada, S. Takamuku, K. Tsumori, S. Takeda, N. Kimura, T. Yamamoto, T. Hori, J. Ohkuma, and T. Sawai, *Mem. Ins. Sci. Ind. Res. Osaka Univ.* **43**, 3 (1986).
 - [13] S. Takeda, K. Tsumori, N. Kimura, Y. Yamamoto, T. Hori, T. Sawai, and J. Ohkuma, *IEEE Trans. Nucl. Sci.* **32**, 3219 (1985).
 - [14] S. Takeda, T. Hori, K. Tsumori, T. Yamamoto, T. Kimura, T. Sawai, J. Ohkuma, S. Takamuku, T. Okada, K. Hayashi, and M. Kawanishi, *IEEE Trans. Nucl. Sci.* **34**, 772 (1987).
 - [15] V. P. Zrellov, M. Klimanova, V. P. Lupiltsev, and J. Ruzicka, *Nucl. Instrum. Methods* **215**, 141 (1983).